Prophecy Variables in Separation Logic

(Extending Iris with Prophecy Variables)

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Reasoning about the correctness of a program

Forward reasoning is often easier and more natural:

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- Reason about how it behaves as it executes

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Strictly forward reasoning is not always good enough!

Reasoning about the current execution step may require:

- Information about past events (this is usual)
- Knowledge of what will happen later in the execution

Remember the past, know the future

Auxiliary/ghost variables store information not present in the program's physical state

History variables [*Owicki & Gries 1976*] (past):

- Record what happened in the execution so far
- Introduced in the context of Hoare logic
- Widely used (modern form: user-defined ghost state)

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Prophecy variables [*Abadi & Lamport 1991*] (future):

- Predict what will happen later in the execution
- Introduced in the context of state machine refinement
- Fairly exotic, (almost) never used for Hoare logic

Motivating example: eager specification

Let us look at a simple coin implementation:

$$\texttt{new\_coin}() \triangleq \{val = \texttt{ref}(nondet\_bool())\}$$

$$\texttt{read\_coin}(c) \triangleq\ !c.val$$

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- A coin is only ever tossed once
- Reading its value always gives the same result

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$$\{\mathsf{True}\}\ \texttt{new\_coin}()\ \{c.\ \exists b.\ \mathsf{Coin}(c, b)\}$$
$$\{\mathsf{Coin}(c, b)\}\ \texttt{read\_coin}(c)\ \{x.\ x = b \wedge \mathsf{Coin}(c, b)\}$$

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$$\mathrm{Coin}(c, b) \triangleq c.val \mapsto b$$

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"Lazy" coin implementation:

$$\texttt{new\_coin}() \triangleq \{val = \texttt{ref}(\text{None})\}$$

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&\quad\ \ \mathrm{Some}(b) \Rightarrow b \\
&\quad \mid \mathrm{None} \quad\ \ \Rightarrow \texttt{let}\, b = \mathit{nondet_bool}(); \\
&\qquad\qquad\qquad\quad\ \ c.\mathit{val} \leftarrow \mathrm{Some}(b);\ b \\
&\quad \texttt{end}
\end{aligned}$$

To keep the same spec we need prophecy variables!!!

Prior work on prophecy variables

Prophecy variables have been used in:

- Verification tools based on reduction [*Sezgin et al. 2010*]
- Temporal logic [*Cook & Koskinen 2011, Lamport & Merz 2017*]

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Only two previous attempts:

- Vafeiadis's thesis [*Vafeiadis 2007*] (informal and flawed)
- Structural approach [*Zhang et al. 2012*] (too limited)

Our contribution: prophecy variables in Hoare logic

We are the first to give a formal account of prophecy variables in Hoare logic!

- Our results are all formalized in the Iris framework
- We also extended VeriFast with prophecy variables
- Useful to prove logical atomicity (RDCSS, HW Queue)

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Presented this morning by Ralf
Prophecies help in case of "future-dependent" LP

Key idea of our approach

We leverage separation logic to easily ensure soundness!!!

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The high-level idea is to use new instruction for:

- Predicting a future observation ($\texttt{let}\ p = \texttt{NewProph}$)
- Realizing such an observation ($\texttt{Resolve}\ p\ \texttt{to}\ v$)

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We leverage separation logic to easily ensure soundness!!!

Principles of prophecy variables in separation logic:

1. The future is ours
   - We model the right to resolve a prophecy as a resource
   - $\mathrm{Proph}_1^{\mathbb{B}}(p, b)$ gives exclusive right to resolve p

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2. We must fulfill our destiny
    - A prophecy can only be resolved to the predicted value
    - A contradiction can be derived if that is not the case

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Prophecy variables are manipulated using ghost code

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`NewProph` (Creates a one-shot prophecy variable p)

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$\{v = b\}$

But we learn that the prophesied and resolved values are equal

Back to the lazy coin example

With the required ghost code the example becomes:

$$\texttt{new\_coin}() \triangleq \{val = \texttt{ref}(\text{None}), p = \texttt{NewProph}\}$$

$$\begin{aligned}\texttt{read\_coin}(c) \triangleq\ & \texttt{match}\,!\,c.val\ \texttt{with} \\ & \quad \text{Some}(b) \Rightarrow b \\ & \mid \text{None} \quad \Rightarrow \texttt{let}\ b = nondet\_bool(); \\ & \qquad\qquad\quad \texttt{Resolve}\ c.p\ \texttt{to}\ b; \\ & \qquad\qquad\quad c.val \leftarrow \text{Some}(b);\ b \\ & \texttt{end}\end{aligned}$$

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The specification can be proved using:

$$\begin{aligned}\text{Coin}(c, b) \triangleq\ & (c.val \mapsto \text{Some}\ b) \vee \\ & (c.val \mapsto \text{None} * \text{Proph}_1^{\mathbb{B}}(c.p, b))\end{aligned}$$

Is the one-shot prophecy mechanism general enough?

Consider the following coin implementation:

$$\texttt{new\_coin}() \triangleq \{val = \texttt{ref}(nondet\_bool())\}$$

$$\texttt{read\_coin}(c) \triangleq\ !\,c.val$$

$$\texttt{toss\_coin}(c) \triangleq c.val \leftarrow nondet\_bool();$$

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What if we want a "clairvoyant" specification?

$$\{\mathsf{True}\}\ \texttt{new\_coin}()\ \{c.\ \exists bs.\ \mathsf{Coin}(c, bs)\}$$

$$\{\mathsf{Coin}(c, bs)\}\ \texttt{read\_coin}(c)\ \{b.\ \exists bs'.\ bs = b :: bs' \wedge \mathsf{Coin}(c, bs)\}$$

$$\{\mathsf{Coin}(c, bs)\}\ \texttt{toss\_coin}(c)\ \{\exists b, bs'.\ bs = b :: bs' \wedge \mathsf{Coin}(c, bs')\}$$

One shot is not enough

Generalization: prophecy a sequence of resolutions!

$\{\texttt{True}\}$

$\quad$ `NewProph`

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`Resolve` p `to` v

$\{\exists bs'.\ bs = v :: bs' \wedge \text{Proph}^{\mathbb{B}}(p, bs')\}$

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Resolving just pops one element

One-shot prophecies can be encoded easily

Back to the clairvoyant coin example

Clairvoyant coin implementation:

$$\texttt{new\_coin}() \triangleq \texttt{let}\, v = \texttt{ref}(\mathit{nondet\_bool}());$$
$$\{\mathit{val} = v, p = \texttt{NewProph}\}$$

$$\texttt{read\_coin}(c) \triangleq\, !\, c.\mathit{val}$$

$$\texttt{toss\_coin}(c) \triangleq \texttt{let}\, r = \mathit{nondet\_bool}();$$
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The specification can be proved using:

$$\mathrm{Coin}(c, bs) \triangleq \exists b, bs'.\, c.val \mapsto b \wedge \mathrm{Proph}^{\mathbb{B}}(p, bs')$$
$$\wedge\ bs = b :: bs'$$

A glimpse at the model of weakest pre

Modified model of weakest preconditions (simplified):

$$
\begin{aligned}
\mathsf{wp}\; e_1\; \{\Phi\} \triangleq\; & \text{if } e_1 \in \mathit{Val} \text{ then } \Phi(e_1) \text{ else} && \text{(return value)} \\
& \forall \sigma_1, \vec{\kappa}_1, \vec{\kappa}_2.\, S(\sigma_1, \vec{\kappa}_1 \mathbin{+\!\!+} \vec{\kappa}_2) \Rrightarrow\!\!\ast \\
& \quad \mathrm{reducible}(e_1, \sigma_1) \wedge && \text{(progress)} \\
& \quad \forall e_2, \sigma_2, \vec{e}_f.\, ((e_1, \sigma_1) \to (e_2, \sigma_2, \vec{e}_f, \vec{\kappa}_1)) \Rrightarrow\!\!\ast \\
& \qquad S(\sigma_2, \vec{\kappa}_2) \ast \mathsf{wp}\; e_2\; \{\Phi\} \ast \mathop{\bigstar}_{e \in \vec{e}_f} \mathsf{wp}\; e\; \{\mathsf{True}\} && \text{(preservation)}
\end{aligned}
$$

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S(\sigma, \vec{\kappa}) \triangleq\; & \boxed{\bullet\, \sigma.1}^{\gamma_{\text{HEAP}}} \ast\; \exists \Pi.\, \boxed{\bullet\, \Pi}^{\gamma_{\text{PROPH}}} \wedge \mathrm{dom}(\Pi) = \sigma.2 \wedge && \text{(state interp.)} \\
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& \quad \mathrm{reducible}(e_1, \sigma_1) \wedge && \text{(progress)}\\
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Observations yet to be made

Reduction now collects "observations"

Wrapping up!

Iris now has support for prophecy variables:

- First formal integration into a program logic
- Useful for logically atomic specifications (Ralf’s talk)
- But that’s not the only application (see François’s talk)

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Things there was no time for:

- Atomic resolution of prophecy variables
- Logically atomic spec for RDCSS and Herlihy-Wing queue
- Erasure theorem (elimination of ghost code)

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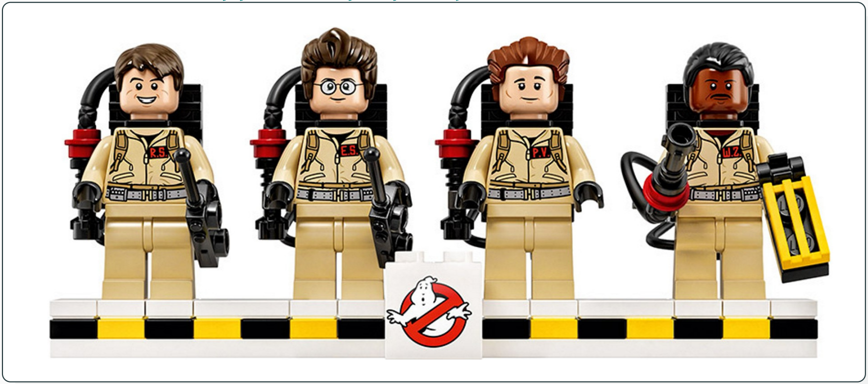
Iris now has support for prophecy variables:



- Erasure theorem (elimination of ghost code)

Thanks! Questions?

(For more details: `https://iris-project.org`)

Model of weakest preconditions in Iris

Encoding of weakest preconditions (simplified):

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$$
S(\sigma) \triangleq \boxed{\bullet\, \sigma}^{\gamma_{\text{HEAP}}} \qquad \text{(state interp.)}
$$

Some intuitions about the involved components:

- The state interpretation holds the state of the physical heap
- View shifts $P \Rrightarrow\!\!\ast\ Q$ allow updates to owned resources
- The actual definition uses the $\triangleright P$ modality to avoid circularity

Operational semantics: head reduction and observations

We extend reduction rules with observations:

$$(\overline{n} + \overline{m}, \sigma) \to_{\mathsf{h}} (\overline{n + m}, \sigma, \epsilon, \epsilon)$$

$$(\texttt{ref}(v), \sigma) \to_{\mathsf{h}} (\ell, \sigma \uplus \{\ell \leftarrow v\}, \epsilon, \epsilon)$$

$$(\ell \leftarrow w, \sigma \uplus \{\ell \leftarrow v\}) \to_{\mathsf{h}} (\ell, \sigma \uplus \{\ell \leftarrow w\}, \epsilon, \epsilon)$$

$$(\texttt{fork}\ \{e\}, \sigma) \to_{\mathsf{h}} ((), \sigma, e :: \epsilon, \epsilon)$$

$$(\texttt{Resolve}\ p\ \texttt{to}\ v, \sigma) \to_{\mathsf{h}} ((), \sigma, \epsilon, (p, v) :: \epsilon)$$

$$(\texttt{NewProph}, \sigma) \to_{\mathsf{h}} (p, \sigma \uplus \{p\}, \epsilon, \epsilon)$$

A couple of remarks:

- Observations are only recorded on resolutions
- State σ now records the prophecy variables in scope

Extension for prophecy variables

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& \forall \{p \leftarrow vs\} \in \Pi.\ vs = \mathrm{filter}(p, \vec{\kappa})
\end{aligned}$$

Some more intuitions about the involved components:

- State interpretation: holds observations yet to be made
- Observations are removed from the list when taking steps

Statement of safety and adequacy

Safety with respect to a (pure) predicate:

$$\mathit{Safe}_\phi(e_1) \triangleq \forall \vec{es}, \sigma, \vec{\kappa}.\ ([e_1], \varnothing) \to_{\mathsf{tp}}^* (e_2 :: \vec{es}, \sigma, \vec{\kappa}) \Rightarrow \mathit{proper}_\phi(e_2, \sigma) \wedge \forall e \in \vec{es}.\ \mathit{proper}_{\mathsf{True}}(e, \sigma)$$

$$\mathit{proper}_\psi(e, \sigma) \triangleq (e \in \mathit{Val} \wedge \psi(e)) \vee \mathrm{reducible}(e, \sigma)$$

Theorem (adequacy). Let e be an expression and ϕ be a (pure) predicate. If wp $e\,\{\phi\}$ is provable then $\mathit{Safe}_\phi(e)$.